Integrating Scientific Phenomena into Mathematics Teaching Using the CBL

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Abstract

The teaching of mathematics can be elevated from the routine recitation of pure forms and abstract relationships to the realistic modeling of scientific phenomena. A system of scientific probes linked to a graphing calculator, called the Calculator Based Laboratory (CBL) system, provides the connection between scientific experimentation, data analysis and mathematical modeling. A 1994 National Science Foundation grant supported interdisciplinary faculty teams to develop mathematics/science modules utilizing the CBL and graphing calculator technology.

Introduction

A teacher's perception of the nature of mathematics greatly influences his or her didactical approach in presenting mathematics to students. If mathematics is the study of pure forms and abstract relationships, then it is sufficient for the classroom lesson to be a blackboard presentation of the algebraic symbolism of x and y's and the equations which join these generic variables. On the other hand, if mathematics is "the queen of sciences," then the purpose of mathematics is to highlight its effectiveness in interpreting patterns and relationships observed in the real world and to draw conclusions about how things work in their natural settings. Of course, mathematics is not a solitary activity or single phase but a process consisting of many stages. These various stages can be described as the discovery, derivation and subsequent application of mathematical forms. The challenge for the teacher is to incorporate all of these phases into classroom teaching and student activities.

An Historical Analogy

The contrasting and apparently divergent aspects of mathematics teaching can be epitomized in the history of mathematics by the philosophy and the work of two notable English mathematicians. On the one hand, there is G. H. Hardy, the pure mathematician of analysis fame, and on the other, Sir Isaac Newton, the preeminent physicist and discoverer of the calculus. In “A Mathematician’s Apology” G. H. Hardy implicitly expresses his disdain for the practical and useful aspects of mathematics. He likens doing mathematics to playing chess, where the enjoyment of executing a beautiful game is its own justification. He advocates the development of mathematics as the study of pure forms needing no other justification than producing works of art and elegance. In contrast, Sir Isaac Newton symbolizes the resourceful applied mathematician, demonstrating the power of mathematics to interpret the results of scientific experimentation. He displays the awesome capability of the mathematician/scientist, able to determine and decipher the intrinsic laws of nature and summarize these in simple mathematical formulations. The legend of the apple falling on Newton’s head represents the wake-up call that reality imposes on us to observe and interpret what is about us.

Teaching Implications

A teacher generally attempts to blend the multiple steps of mathematics into classroom presentations. The entire mathematical process of discovery, derivation and application deserves to be presented to and experienced by students. However, the discovery and the application phases usually get short shrift in the classroom setting due if the teacher is distant and isolated from the problem source and context. The initial thrill of confronting the source of the problem and formulating the structures and concepts to understand the problem is what is missing. An experienced or astute teacher will try to create the air of inquiry and discovery by using analogies or by relating stories of famous situations to portray the invention of mathematical theories. Nevertheless, does he or she have the resources or time to simulate the original situation generating the problem at hand?

Without the original problem exposed and explained, the subsequent application of equations becomes an artificial exercise. The proposed mathematical model claiming to fit the setting and providing the solution to the problem now becomes an isolated and often meaningless entity, especially from the students point of view. Neither is the typical mathematics teacher generally trained to be a scientist. Furthermore, the curriculum does not allow time for mathematical instruction to include lengthy observational studies or scientific experimentation. It is more convenient for a mathematics teacher to concentrate on working with the abstract mathematical forms, separate from and devoid of its natural setting. What gets lost in the mathematics lesson is what originally motivated the mathematician to generate the effective and elegant mathematical solution.

Graphing Calculator Technology

Recent advances in graphing calculator technology have presented mathematics teachers the opportunity to motivate the many mathematical forms they teach in the classroom, whether linear, quadratic, inverse, logarithmic or exponential. Since 1988, graphing calculators have been used as portable, handheld calculation instruments with the capability of displaying the equations and graphs of a diverse range of functions, especially those cited above. The technology now allows mathematics teachers the ability to initiate the process of discovery, produce the scientific phenomena related to the problem and generate the appropriate data to test the proposed mathematical model. The key to this new window to scientific data is afforded by a system of scientific probes or sensors called the Calculator Based Laboratory (CBL) system. These probes were already being used in science classrooms and laboratories, produced by such companies as Pasco, Inc., but the systems were computer-based. In 1993, an arrangement between Texas Instruments, Inc. and the Vernier Company adapted these scientific probes for use with graphing calculators, providing mathematics teachers instant and portable access to scientific experimentation.

Presently there are over 30 scientific probes able to measure such physical, chemical or biological variables as temperature, light intensity, velocity, force, sound pressure, voltage, magnetism, pH, color intensity, and the amount of oxygen or carbon dioxide in solutions. These have opened the entire world of scientific experimentation to the classroom setting without requiring intricate or elaborate laboratory instrumentation. The probes are linked to the data collection device called the CBL, which in turn is connected to the graphing calculator, within which are stored the program operating the data collection and from which data may be stored, viewed and graphed.

A Sampling of CBL Demonstrations

Suppose a mathematics teacher is discussing the concept of linear functions, easily formulated on the board in y = mx + b form. What real-life examples of linear equations are readily accessible beyond resorting to textbook examples. Yet simply setting up a motion detector to the CBL system, a teacher is able to demonstrate linear motion concretely, by having students walk paths of motion that create linear graphs. In these classroom-walking exercises, students kinesthetically learn that constant motion translate into lines, that horizontal lines are produced by standing still (in time) and that straight lines with steeper slopes correspond to faster (constant) velocities. An instructive exercise for students is to simulate a predetermined graph (Figure 1) by shifting walking speeds as the motion detector is registering the action.

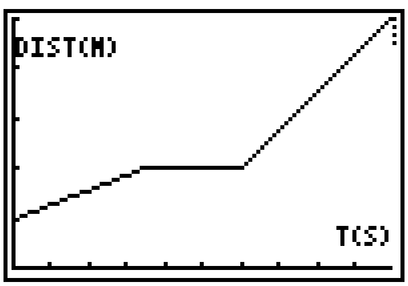

Figure 1

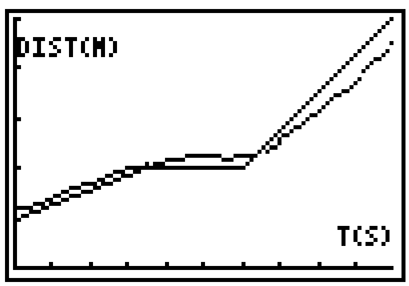

Figure 2

In these graphs, time is measured on the x-axis in units of seconds with each tick marking out 10 seconds, while the distance of the walker from the detector appears on the y-axis in units of 1/2 meters. The student attempting to simulate this graph must gauge his or her motion while viewing the graph to be modeled. An actual student's attempt at walking this graph appears in Figure 2. Students quickly learn how to create this real-world model of motion by varying their walking speeds to affect the linear slope.

The Calculator Based Ranger

In the most recent technology, the original motion detector has been replaced by the CBR, the Calculator Based Ranger, which combines the original motion detector and the CBL data collector into one device. The CBR only needs to be connected to a graphing calculator for complete operation. One of the latest models of Texas Instrument graphing calculators is the TI-83 Plus, which has the CBL/CBR programs already contained in its program memory, accessible through an applications key. An instructive example using the CBL/CBR program is capturing the motion of a bouncing basketball.

Figure 3

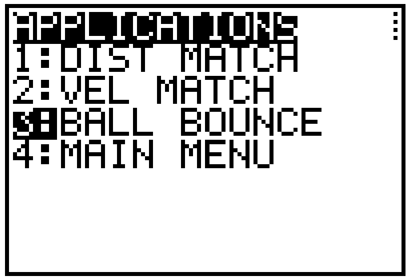

Figure 4

The CBR is held above the ball and the trigger is pressed an instant before a basketball is released, creating a graph that appears below in Figure 5 on the left. Many questions about nonlinear motion naturally arise in viewing this graph, such as: What is the mathematical form or equation of one arc? What is an equation of the curve passing through the peaks of each of the arcs? Interpret the velocity function of this ball-bouncing graph? The velocity graph appears below on the right in Figure 6, and can be an eye opener for students, for it consists of abruptly interrupted periods of linear velocities, which needs to be explained in the context of constant acceleration under gravity. Hence, Sir Isaac Newton reappears!

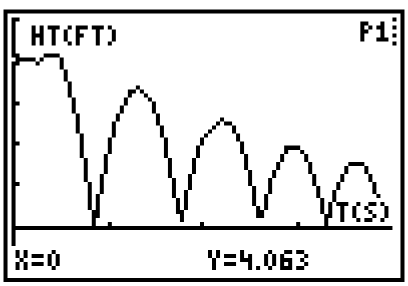

Figure 5

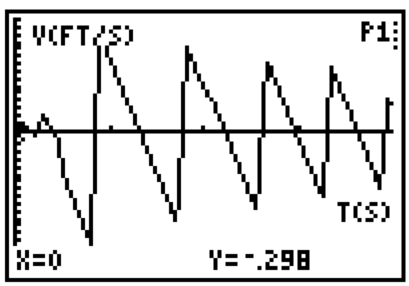

Figure 6

This ball bouncing demonstration can be recast into a classroom activity involving groups of students working together with CBRs and graphing calculators to determine the coefficients of restitution for several bouncing objects, such as a basketball, a softball, a tennis ball, a golf ball and a superball. The students must determine the order that these balls rebound from highest to lowest, first by visual inspection and then by more precise motion detector measurements. Each student in a group should be given the chance to do a CBR recording for one of the different balls. In each case, the student is attempting to determine the rebound rate r in the formula for the height of the ball, $h = ar^n$, for $0 < r < 1$. The students can determine how well their visual estimations of ball rebound compare with the CBR measurements.

Measuring Temperature

A simple probe that comes with the CBL kit is the temperature sensor. What kinds of graphs occur when temperatures are recorded over time intervals? Consider the following experiment, easily conducted in a classroom setting. The teacher holds a hot cup of tea or coffee and tells the class that after 20 seconds, she will insert the temperature probe into the cup of hot beverage. After the probe is in the cup for 40 seconds, she will remove the probe back into the air and keep it there for the remaining 40 seconds. She asks the class to attempt a graph of the fluctuating temperature, being as precise about actually temperature values (at key points) as possible. What does this graph look like and what does it teach students about warming and cooling objects?

The graph of this experiment appears below in Figure 7. An examination of it shows three major phases, a period of constant value for 20 seconds at ambient room temperature, measured at about 27.5^o C, an increase in temperature to a peak value of about 63^o C in about 40 seconds (see Figure 8), and then a final temperature retreat as the probe is returned to the air.

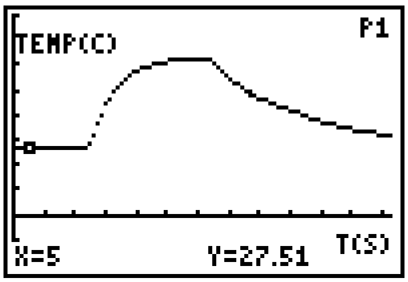

Figure 7

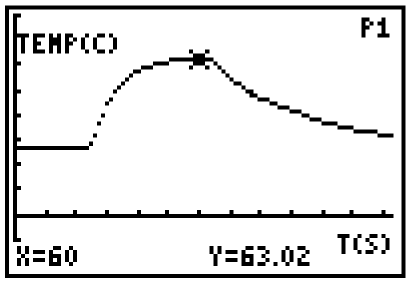

Figure 8

The graphs that students propose can be reviewed to see it these detect the basic concave/convex feature of the different parts of the curve, a part displaying a variation of exponential growth and then that of exponential decay. Beyond the initial constant portion of the graph, do students understand the concave downward and then concave upward nature of the curve? Have they drawn linear graphs for the increasing and then decreasing portions of the graph? A linear graph with negative slope, of course, would continue beyond the limits of room temperature. Further investigations into the falling temperature portion of the graph shows that the slopes themselves form a linear pattern. (The graphing calculator actually records the temperatures at about one-second intervals in convenient data lists. Adjacent points can be used to calculate slopes over desired intervals.) The linear nature of these varying slopes confirms the fact that the temperature of the cooling portion obeys a first order differential equation, called Newton’s Law of Cooling, which states that the rate of change of temperature is proportional to the difference between the temperature of the probe and room temperature. This experiment can be done in a mathematical analysis class covering exponential functions, even before the introduction of the calculus. A real challenge is to generate the piecewise function that simulates the graph of this curve.

The CBL is portable enough to be set up outside to record outdoor temperatures every hour over a 24-hour period. Will this graph show a basic sinusoidal pattern or is does the fluctuating temperature behave in a more complicated nature? When does the minimum and maximum temperatures of the day/night occur? These real-life questions deserve experimental answers rather than simplistic textbook responses. It engenders natural questions that beg for solution.

The cooling of coffee or tea has actually been studied quite elaborately in a controlled laboratory setting. See “On Cooling Tea and Coffee," by W. G. Rees, appearing in Am. J. Phys. 56 (5), May 1988. Many of the experimental results become the source of classroom questions that can be posed about differing cooling conditions. A famous differential equation question involves a summit meeting between a President and Prime Minister. During a tea or coffee break, both are poured equal amount of hot beverage in similar cups. The President immediately adds a fixed amount of cold cream but the Prime Minister waits five minutes before putting the same portion of cream in his cup. Both dignitaries wait a total of 10 minutes before drinking. Which drinks the cooler coffee?

Two CBL with temperature probes can be set up to experimentally solve this question. Even doing it theoretically is assisted by the fact that actually temperatures of coffee and cream can be calibrated to a realistic setup of initial and intermediate conditions for paper and pencil computations. If this question is too difficult for a class who have not encountered the calculus yet, then alternate cooling questions can be posed? The previously mentioned physics article provides a teacher with enough empirical results to surmise the answers. Some typical cooling questions are: Hot coffee or tea cools quickest in which type of container, one made up of paper, Styrofoam, ceramic, glass or metal? How much difference does a lid provide to lessen the cooling effect? Does cooling get enhanced using different types of liquid? These and a multitude of other cooling questions stir the curiosity of students, motivating them to become amateur scientists or budding mathematicians confronting a myriad of simple queries to investigate.

Measuring Sound

The sound probe is a simple microphone registering fluctuations in sound pressure. Trying to measure the frequency of a tuning fork is a standard classroom demonstration. However, the CBL can extend this demonstration to a host of other sounds that are simple to create and display. For instance, what sound pattern occurs when a plastic water or cola bottle is blown? What note is this? The graphs below in Figures 7 and 8 show a real curve obtained by blowing cross the lip of a plastic soda bottle. One trough (minimum) has value X = .006304, while an adjacent one is X = .010928 for a wavelength of $\Delta X = .004624 = \lambda$ and frequency f $= 1/\lambda = 216.3$ cycles per second, which is very close to an A note at 220 hertz.

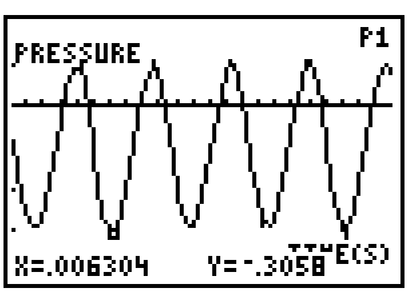

Figure 7

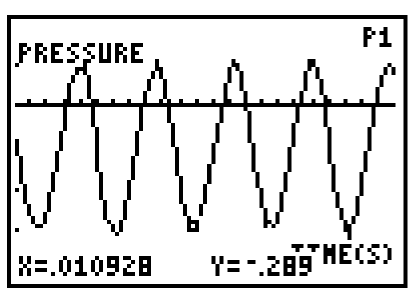

Figure 8

As students gain mastery at producing bottle sounds and calculating their frequencies and corresponding musical notes, an interesting question to toss out is: If a bottle is half filled with water how does the frequency of the sound change? In fact, does this frequency change as more water is added to the bottle? Is it a linear function based on volume of water or height of the water in the bottle? All of these are ripe for CBL experimentation.

Many other simple questions about sound can be posed and answered. Teachers can display the snap of a finger or what a high-pitched whistle look likes graphically. The graph below in Figure 9 shows what whistling through one's teeth looks like, very reminiscent of the phenomenon of beats, with the high pitch showing as a pattern of short waves. However, these shorter more frequent waves appears to be enveloped by a longer wave, the creation of which needs to be analyzed. CBL sound experiments of a more scientific nature include the measuring of the speed of sound as it traverses a tube, which needs only to be a few feet long.

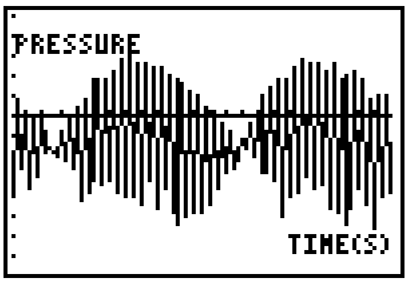

Figure 9

Measuring Light Intensity
Another useful probe is the light intensity-measuring device, the light probe, for short. It also is inserted into one of the external ports of the CBL. A simple demonstration is to point this device at an incandescent or fluorescent light bulb and view the graph of the energy levels generated. Two such graphs appear below, with the graph in Figure 11 on the right that of the light intensity from a bank of fluorescent tubes. A more scientific experiment is measuring the light intensity of a light bulb as a function of the distance of the probe from the light source. What is the nature of this relationship?

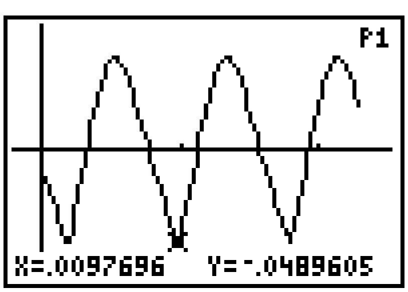

Figure 10

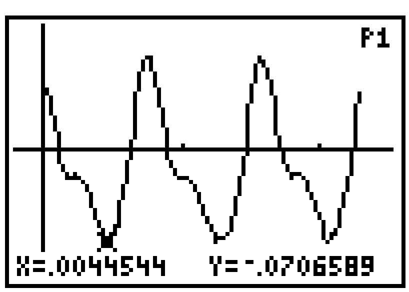

Figure 11

Since 1994 many manuals have been produced and printed, delineating the myriad of scientific experiments available for classroom adaptation. See Brueningsen, etc. In 1994 the author of this article, along with colleagues at Middlesex County College (MCC), Edison NJ, and local school districts, undertook a curriculum development project supported by the

National Science Foundation (DUE-9454604) to develop interdisciplinary math/science modules employing the CBL and graphing calculator technology. A sample of these modules can be accessed through the MCC mathematics website at http//:www.njin.net/mcc_math/CBL/cmain.htm. Teams of four instructors, pairing two from mathematics and two from a science discipline, from both collegiate and secondary school ranks, worked together to produce integrated mathematics and science classroom lessons. The physics foursome had the assistance of scientists from the Princeton Plasma Physics Laboratory, the biology team was connected to the NJ Marine Science Consortium, and the chemistry team obtained input from the Merck Science Education Institute. This unique cooperative effort of academic and industrial partners displayed the collaborative planning needed to make mathematics a genuine realistic endeavor in problem solving and scientific exploration. The gap between abstract mathematics and real-world applications had been noticeably bridged by this collaborative venture.

Bibliography

Brueningsen, Chris, et al. Exploring Physics and Math with the CBL System. Texas Instruments Incorporated, 1994.

________________, et al. Math and Science in Motion: Activities for Middle School. Texas Instruments Incorporated Exploration Series, 1997.

________________, et al. Real-World Math with the CBL System: Activities for the TI-83 and TI-83-Plus. Texas Instruments Incorporated Exploration Series, 1999.

Hardy, G.H. "A Mathematician's Apology." The World of Mathematics, Volume IV, ed. John R. Newman, Simon & Schuster, New York, 1956.

Rees, W.G. & Viney, C. "On cooling tea and coffee." Am. J. Phys. 56 (5), May 1988.